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The Strange Attractor of Chaotic Voltage Oscillation in Water-Oil-Water Liquid Membrane Containing Cationic Surfactant

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A strange attractor of chaotic voltage oscillation caused by micelle formation of cationic surfactant in the water-oil-water liquid membrane was found. The topology of the strange attractor is the mirror image of the Rössler's attractor. The correlation dimension of the attractor is 2.4.

Spontaneous periodic and quasiperiodic variations of interfacial tension and electrical voltages between immisible phases have been reported by Dupeyrat,[1-3] Yoshikawa[4] and are gathering attention as a model of spontaneous voltage oscillations of cell aggregates. Yoshikawa *et al* have found that the liquid membrane, the water-oil-water which contains surfactant, showed stable spontaneous voltage oscillations across oil and water phases.[5,6] We tested the same system and have found bifurcation of single period - double period - chaotic time series of voltages.

The experimental setup consists of a U-shape glass tube of 13 mm in diameter which contains a 4 ml nitrobenzene solution of picric acid at the bottom and two 5 ml aqueous solutions in both side of U-arms. One of the aqueous solutions contains 5 mM cationic surfactant, cetyltrimethylammonium bromide(CTAB), and 1.5 M ethanol. The other arm contains 0.1 M glucose. Voltages between water phases were measured with Ag/AgCl electrodes at the temperature of 30 ℃ and stored in the disk memories through an A/D converter. The behaviour of voltage oscillations at various concentrations of the picric acid between 0.6 mM and 1.5 mM was tested.

The typical chaotic time series of voltage across the liquid membrane, are shown in Fig.1. The characteristics of voltage time series at various concentrations of picric acid are summerized in Table 1. Oscillations were single period at higher concentrations than 1.0 mM. Duration between pulses decreased with the picric acid concentration. Double period oscillation were found at 0.8 mM picric acid. Around 0.6

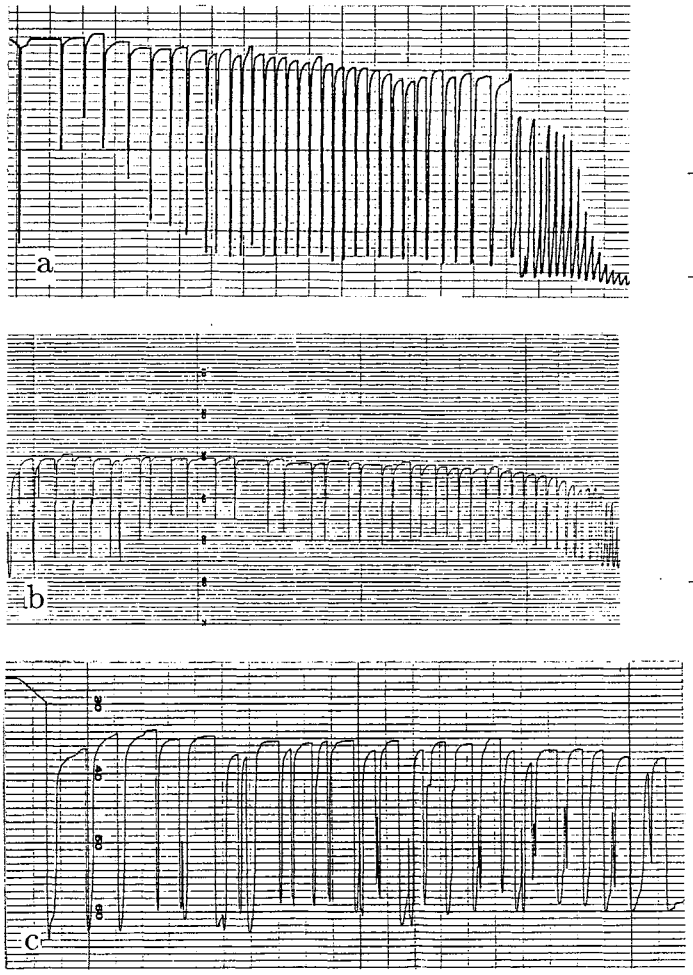

Fig.1. Voltage time series across liquid membrane. Concentrations of picric acid are a: 0.90 mM (single period), b: 0.80 mM (double period), c: 0.60 mM (chaotic).

Table 1. Characteristics of voltage time series observed across liquid membrane containing cationic surfactant

picric acid concentration /mM	pulse duration /sec	height /mv	induction time of first pulse /min
0.60	chaotic	240	51
0.64	chaotic	215	67
0.80	72, 132	188	66
0.90	34.8	184	9
1.10	33.6	172	8
1.20	31.2	160	8
1.30	30.0	192	8
1.40	24.0	156	6
1.50	22.8	140	6

mM picric acid the oscillation showed chaotic behavior. In the course of picric acid concentration the oscillation showed bifurcations at 0.8 mM and 0.6 mM. No voltage oscillation was found at a lower picric acid concentration than 0.6 mM.

The embedding theorem[7-9] is used to reconstruct an attractor of the dynamical system from the time series of voltage. The chaotic voltage time series shown in Fig.1c gives the attractor which is drawn in Fig.2.

Poincaré sections around the axis which is selected to go through the circle of the attractor were drawn in Fig.3. The orbit stretches between 180° and 320° sections an folds between 340° and 60° sections. The model drawing of the strange attractor is shown in Fig.4. The attractor is a simple stretching-folding strange attractor and has the topology which forms a mirror image of Rössler's attractor: $dx/dt = -y-z$, $dy/dt = x + 0.2y$, $dz/dt = 0.2 + z(x-5)$.[10]

The D_2 fractal dimension, the correlation dimension, of the strange attractor was calculated by the box counting method[11] and shown in Fig.5. The number of points within the ν-dimensional sphere of radius ε is

$$C(\varepsilon) = \frac{1}{N^2}\sum_{i,j}^{N} H(\varepsilon - |x_i - x_j|),$$

where H is Heaviside function, N is the total number of points in the attractor, and x_i is the coordinate of i-th point. The attractor was formed by 923 points. Randomly sampled 100 points out of them were used for the calculation of correlation integral $C(\varepsilon)$. $C(\varepsilon)$ is shown in Fig.5 where absissa shows the logarithm of radius ε and the ordinate shows the logarithm of the number of points $C(\varepsilon)$. The correlation

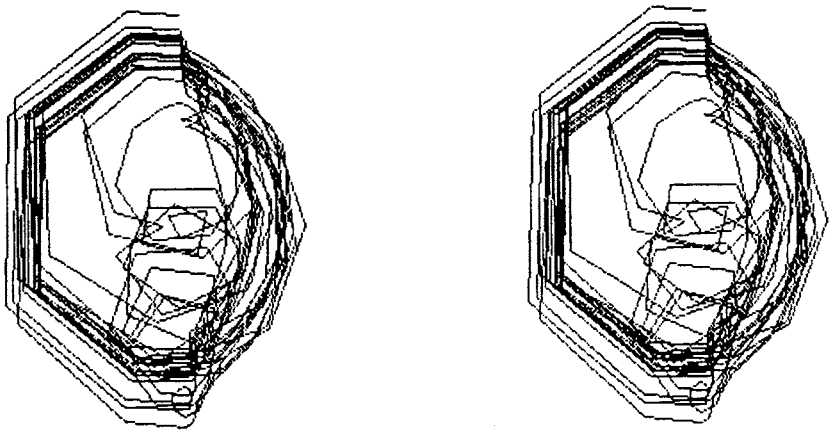

Fig.2. Strange attractor embedded in $\nu = 3$ dimensional space. Reconstracted from chaotic voltage time sequence of Fig.1c. (anticlockwise)

dimension was obtained as the limit of each ν-dimensional slope which converged to the value of 2.4 which is greater than 2 and less than 3. In the case of 0.64 mM picric acid concentration, the attractor forms a part of Fig.3. All other non-chaotic attractor forms much simpler part of Fig.3.

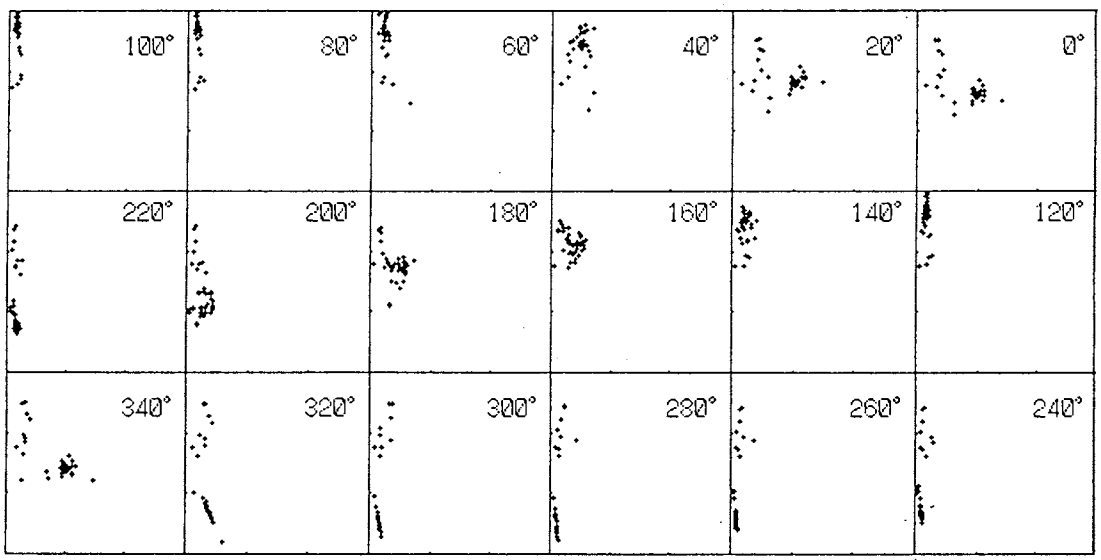

Fig.3. Poincaré sections of the strange attractor.

In conclusion, we found that the time series of voltage oscillations across liquid membrane which contains cationic surfactant showed bifurcations among single period - double period - chaotic oscillations. The attractor which is reconstructed from the chaotic oscillation was a simple strange attractor whose topology is the mirror image of Rössler's model. The correlation fractal dimension D_2 is 2.4.

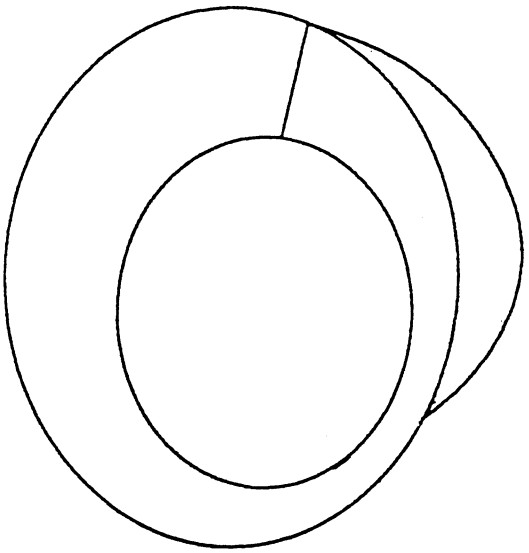

Fig.4. Model drawing of the strange attractor.

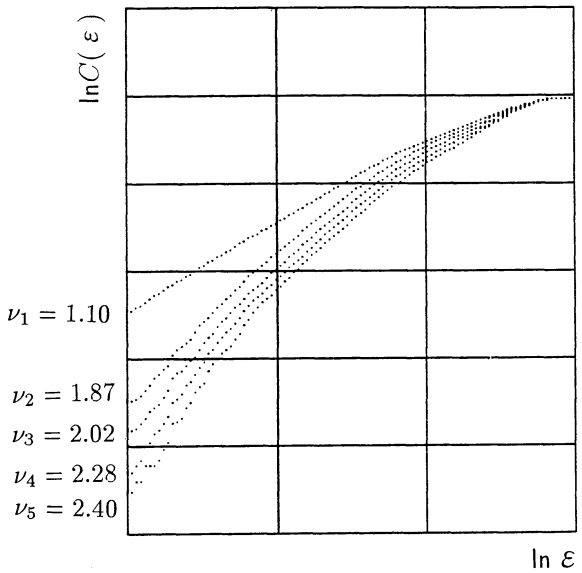

Fig.5. Correlation dimension of the strange attractor : 2.4. (see text).

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